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Forming of metal ring disc and studying of mechanical properties

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Abstract— In developing the application, and technological problems of the ax symmetric shape-changing of metals, using the results of solving the problem of determining the stress-strain state of a ring disk and a thick-walled tubes at large plastic deformations is mentioned. These issues include the definition of finite quadratic form sizes and products, and mechanical properties of the material changes due to deformation at a given value and patterns of changes in the external load. Current methods for analyzing such problems is based on a schematic of the process of forming, in which the problem reduced to giving the equations that characterize the plastic state, convenient for forms of mathematical calculations in order to obtain closed analytical solutions. To do this, consider the problem of statically determinate, so that it becomes possible to determine the stress equations without consideration of the relationship between stress and strain (strain rates).

Index Terms— Yield criterion, Tresca yield condition, Metal ring discs, Thick-walled tubes, Stress-strain state, Mechanical properties, Plasticity, Internal pressure, Von Mises yield criterion.

1 Introduction

CURRENT methods for analyzing such problems is based on a schematic of the process of forming, in which the problem is reduced to give the equations that characterize the plastic state, convenient for forms of mathematical calculations in order to obtain closed analytical solutions. To do this, consider the problem of statically determinate, so that it becomes possible to determine the stress equations without consideration of the relationship between stress and strain (strain rates).

Current state analysis of ax symmetric problems is based on the pioneering work [1, 8], which examined the plastic state of thick-walled tubes of infinite length under uniform internal pressure. In these studies it is assumed, according to which tube cross-sections remain plane in the shape changing, based on which it becomes possible to determine the stress of the joint solution of the equilibrium equation and the condition of plasticity for plane-stress or to the plane-strain conditions. From these solutions it follows that the limiting value of internal pressure relative diameter of the pipe in a plastic state, in the first case approaches 2.963, while the second - to 2.72. In fact, in both cases, solved the plane problem in polar coordinates with different forms of writing the conditions of plasticity. The results obtained are somewhat contradictory, because of the difficulties the assumption that for the same value of the internal pressure of the absence of axial compressive stress, increases the relative size of the plastic region. In addition, these solutions, it is difficult to establish the relationship between the coordinates of the elements being considered in the initial and deformed states, so the definition of the ultimate shape and size of the product becomes an impossible task.

In another class of works [1, 2, 3] the elastic-plastic state of thin plates with a circular cutout under internal pressure. In particular, in [1] that in the limiting condition on the inner contour of the plate, radial and circumferential stresses, respectively, reaching values $\sigma_\rho = -2\sigma_s/\sqrt{3}$, $\sigma_\theta = -\sigma_s/\sqrt{3}$, ,and on the basis of equations to the stress and strain increments is concluded that the rate (increase) axial strain, and the relative radius $d\varepsilon_z \to \infty$ of the plastic region approaches 1.75 when using the von Mises yield criterion, and to 1.65 when using the Tresca yield condition, the Saint-Venant . In [2, 3] attempted to obtain a more realistic result for the axial component of strain. It is shown that the limiting value of the internal pressure of the relative thickness of the inner contour approaches 3.61. The unreality of the results, the authors believe, is due to a convention schematic process of forming.

If you work in a first direction contradictory results involves considering the plane stress state based on the hypothesis of plane sections, in the second case, unrealistic results due to the erroneous assumption that is making, is associated to the mechanics of forming.

It therefore seems appropriate and relevant mathematical problems correct formulation of forming an annular disk of constant thickness of a material with specific mechanical properties in a large plastic deformation.

We consider here forming a limiting state, taking into account the interrelated changes in the axial component of strain and strain hardening, and the following tasks:

A. Determination of the maximum relative diameter disc in a plastic state;
B. Identify linkages between final and initial shapes and sizes of the product.

2 Basic Equations and General Method for Solution

Consider a circular disc of thickness H_0 inner radius r_0 and outer R_0, loaded by internal pressure p. We define the stress-strain

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towels of the disk, without imposing restrictions on the values of deformation. In contrast to the well-known papers [1, 3, 5] assume that the disk consists of a set of annular plates of the size $2r_0$, $2R_0$ and thickness h_0, which can be freely deformed in the radial and axial structure Trends in maintaining the continuity of material and the absence of force interaction between the plates. Obviously, such a model is the most realistic display of the disk under internal pressure and the absence of an axial compressive force.

We adopt a cylindrical coordinate system ρ, θ, z whose plane is z=0 the mean plane of the plates, and the axis z perpendicular to this plane. It is assumed that the plates are loaded by external forces, parallel to the median plane, and the characteristic size of the plates $h_0/(R_0 - r_0) \ll 1$. In such conditions the radial, axial and circumferential directions are principal directions of stress and strain and stress state which is realized in such an environment, described as a generalized plane stressed state [1].

So the problem of deformation of the annular disc is reduced to the analysis of forming multiple thin-walled circular plates deformed in plane stress under internal pressure. To achieve the objectives we consider the basic equations that characterize the plastic state:

Mises yield condition (1), the intensity of deformation (2), the condition of constant volume (3), the relationship between stress and strain increments (4), in which - the intensity increment of deformation, defined by (5).

$$\sigma_s = \left\{\frac{1}{2}\left[(\sigma_\rho - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_\rho)^2\right]\right\} 1/2 \qquad (1)$$

$$\varepsilon_i = \left\{\frac{1}{2}\left[(\varepsilon_\rho - \varepsilon_\theta)^2 + (\varepsilon_\theta - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_\rho)^2\right]\right\}^{1/2} \qquad (2)$$

$$\varepsilon_\rho + \varepsilon_\theta + \varepsilon_z = 0 \qquad (3)$$

$$\frac{d\varepsilon_\rho - d\varepsilon_\theta}{\sigma_\rho - \sigma_\theta} = \frac{d\varepsilon_\theta - d\varepsilon_z}{\sigma_\theta - \sigma_z} = \frac{d\varepsilon_z - d\varepsilon_\rho}{\sigma_z - \sigma_\rho} = \frac{3}{2}\frac{d\varepsilon_i}{\sigma_s} \qquad (4)$$

$$d\varepsilon_i = \left\{\frac{2}{9}\left[(d\varepsilon_\rho - d\varepsilon_\theta)^2 + (d\varepsilon_\theta - d\varepsilon_z)^2 + (d\varepsilon_z - d\varepsilon_\rho)^2\right]\right\}^{1/2} \qquad (5)$$

The relationship between tension stress and equivalent strain introduced in the form of a power function (6). In (6) K and n- the parameters of strain hardening, depending on the mechanical properties of deformable material relations: $K = \sigma_b e^n n^{-n}$, $n = ln(1+\delta)$, $(\sigma_b, \delta$ respectively, the stress the tensile strength and uniform elongation of the relative value of the linear tension).

$$\sigma_i = k\varepsilon_i^n$$
(6)

The equation of equilibrium of an element axially loaded circular plate with the availability of axial deformation of the form (7).

Equation (7) contains three unknowns are a function of coordinates. Additional use of the equations of plasticity (1) does not make the problem statically determinate. To find the stress field requires the use of constraint equations (4).

$$\rho\frac{d_\sigma}{d\rho} + \sigma_\rho\left(1 + \frac{\rho d_s}{s d_\rho}\right) - \sigma_\theta = 0 \qquad (7)$$

Equation (7) with the plasticity condition (1) takes the form (8), which does not have a closed analytical solution for $\sigma_s = var$, so that it becomes impossible to obtain expressions for the stresses interdependent change in thickness of sheet metal and strain hardening.

$$\rho\frac{d\sigma_\rho}{d\rho} + \frac{2\sigma_s^2}{\pm\sqrt{4\sigma_\rho^2 - 3\sigma_\rho^2}} = 0 \qquad (8)$$

Following the method described in the work and representing the increment (rate) of the principal strains in the deviatory plane cylinder plasticity (π-plane) in the trigonometric form (9), equation (8) is displayed on this plane in the form of a simple differential relation (10), $d\sigma_\rho = \sigma_s d\varepsilon_i$

$$d\varepsilon_\rho = d\varepsilon_i \cos\varphi, d\varepsilon_\theta = d\varepsilon_i \cos\left(\varphi + \frac{2}{3}\pi\right), d\varepsilon_z = d\varepsilon_i \cos(\varphi + \frac{4}{3}\pi)$$
(9)

$$d\sigma_\rho = \sigma_s d\varepsilon_i \qquad (10)$$

The main problem in solving (10) is that the intensity increment of deformation (5) and increase the intensity, determined by differentiating the relation (2) are not equal, so the integration of expressions of the form $\int \varepsilon_i d\varepsilon_i$ can only be done in certain ways deformations.

From the analysis it follows that the integration of (10) for a given law of change of tension stress on the current value of the effective deformation is only possible with a constant ratio $\sigma_\rho/\sigma_\theta$ of the element to the work piece in the process of forming. From this constant should be constant parameter φ, which means accepting the assumptions of the linear nature of the accumulation of strain in the deviatory plane cylinder plasticity (π-plane).

Integrating (10) with a view of the above, after simple transformations we obtain (11)

$$\varepsilon_i^{n+1} - \varepsilon_{kp}^{n+1} = (1+n)\varepsilon_i^n \frac{2}{\sqrt{3}} cos(\varphi + \pi/6)$$

(11)

The limits of the parameter φ in the problem set as a function of applied load. When a small amount of internal pressure annular plates are in the elastic state and stresses are determined from the known solutions of the elastic problem [1,4]. With the increase of internal pressure to the initial critical value, equal to $\sigma_s/\sqrt{3}$, the internal loop appear first plastic deformation, while the equation $|\sigma_\rho| = |\sigma_\theta| = \sigma_s/\sqrt{3}$. With increasing pressure the plastic deformations are distributed in the direction of the outer contour and the limiting value of the internal pressure relative diameter of the disk covered by plastic deformation reaches its maximum value.

In this range of the parameter φ vector function ε whose modulus is numerically equal to the equivalent strain (2) [6,7], it is perpendicular to the coordinate axes ε_θ and ε_z , resulting in the increment and the strain components along these axes are equal to zero.

Of dependency that the carrying capacity of plates is exhausted, when the radial compressive stress on the inner contour of the absolute value reaches a value $2\sigma_s/\sqrt{3}$ at which $\varphi = 5\pi/6$ h. At the boundary of the elastic-plastic deformation of the axial component becomes zero, so that this boundary condition$|\sigma_\rho| = |\sigma_\theta| = \sigma_s/\sqrt{3}$. Consequently, all kinds of strains, which in principle can be realized in the limit state located in the deviatory plane as a sector with central angle$\varphi = \pi/3$. If the parameter φ is equal to the negative direction of the axisε_ρ, ε_ρ- compression deformation in the radial direction, ε_θ and ε_z- tensile strain, which is numerically equal$\varepsilon_\rho/2$.

If this parameter φ coincides with the direction$5\pi/6$, then $\varepsilon_\theta = 0$ but also $\varepsilon_\rho$and $\varepsilon_z$are equal in value and opposite in sign, that is, there is a net shift in plane strain or plane(ρ, z). If this parameterφ coincides with the direction$7\pi/6$, then $\varepsilon_\theta = 0$ but also ε_ρ and $\varepsilon_z$are equal in value and opposite in sign, that is, there is a net shift in plane strain or plane(ρ, θ).

From the relation (11) are easily determined strain components (12) with$\varepsilon_0 = 0$.

$$\varepsilon_\rho = \frac{1+n}{2}(1 + \cos 2\varphi - \frac{\sqrt{3}}{3}\sin 2\varphi)$$

(12)

$$\varepsilon_\theta = -\frac{1+n}{2}(\cos 2\varphi + \frac{\sqrt{3}}{3}\sin 2\varphi$$

$$\varepsilon_z = -\frac{1+n}{2}(1 - \frac{2\sqrt{3}}{3}\sin 2\varphi)$$

Establish the relationship between the parameter φ and the coordinateρ of the element in the deformed state. For this purpose, following the procedure set forth in section (3) [9-10]

$$\frac{d\rho}{\rho} = -(1+n)\left(\sin\varphi\cos\varphi + \frac{\sqrt{3}}{2}\sin^2\varphi + \frac{\sqrt{3}}{6}\cos^2\varphi\right)d\varphi$$

(13)

After integrating (13) reduces to

$$\ln\rho = -(1+n)(\frac{\sqrt{3}}{3}\varphi - \frac{1}{4}\cos 2\varphi - \frac{\sqrt{3}}{12}\sin 2\varphi) + C. \quad (14)$$

The integration constant in (14) is the boundary condition according to which at $\varphi = \frac{7}{6}\pi$; $\rho = R$ (R- outer radius of the plastic region). From the boundary condition equation (14) takes the form:

$$\frac{\rho}{R} = \exp\{(1+n)[-\frac{1}{4} + \frac{\sqrt{3}}{3}(\varphi - \frac{7\pi}{6}) + \frac{1}{4}\cos 2\varphi + \frac{\sqrt{3}}{12}\sin 2\varphi]\} \quad (15)$$

In deriving the relation (15) takes into account that the effective strain is always a positive scalar quantity.

Substituting into (15) the value $\varphi = {}^{5\pi}/_{6}$ at which the internal pressure reaches the limit, we obtain the relative maximum diameter of the plastic region

$$\frac{R}{\rho_0} = \exp[(1+n)(\frac{1}{4} + \frac{\sqrt{3}\pi}{9})] \approx \exp[(1+n)(0{,}854)], \quad (16)$$

Where ρ the radius of the inner contour of the deformed state. When$n = 0$, the limit value R/ρ_0 tends to 2.35, which is much smaller than the result of [1, 2, and 3]. The resulting solution, as a parametric solution of the problem, completely determines the stress-strain state of circular plates with allowance for the interrelated changes in axial strain components and strain hardening.

To determine the final shape and size of the product when the limiting value of internal pressure consider the distribution of circumferential and axial strains. In the presented analysis of the increment of the strain components referred to the current deformed state, and their summation leads to logarithmic deformations that satisfy the condition of constant volume:

$$\varepsilon_\rho = \ln\frac{d\rho}{dr}; \quad \varepsilon_\theta = \ln\frac{\rho}{r}; \quad \varepsilon_z = \ln\frac{h}{h_0}$$

(17)

Where$\rho, h$and r, h_0 the current position and thickness of the element of the plate in the deformed and initial states, respectively.

From equation (12) and (17) can be easily installed and coordinates the relationship between the thickness of the current and original states.

$$\rho/r = \exp\left[\frac{1+n}{2}\left(\cos 2\varphi + \frac{\sqrt{3}}{3}\sin 2\varphi\right)\right];$$

$$h/h_0 = \exp\left[\frac{1+n}{2}\left(1 - \frac{2\sqrt{3}}{3}\sin 2\varphi\right)\right].$$

(18)

In particular, the internal circuit of $\varphi = \pi$; $(|\sigma_\rho| = |\sigma_s)$

$$\rho_0/r_0 = h/h_0 = \exp\frac{1}{2}(1+n)$$

(19)

For a perfectly rigid-plastic model of a deformable material$(n = 0)$, the relative thickness of the inner edge of 1.648 Comparing the solution obtained from the second relation (9), we easily see that the limiting value of internal pressure equal $2\sigma_s/\sqrt{3}$ and $d\epsilon_z$ can not tend to infinity, since in this case both the numerator of this function is zero.

On the other hand, from the mechanics of forming that when the value of internal pressure equal σ_s to the module of the vector function can not be greater than one, at that$\varphi = \pi$. Consequently, the maximum value the axial component of the deformation tends to 0.5, while the relative thickness of the plate - to 1.648, which corresponds to the values realized in the experiments. Stress distribution in elastic and plastic regions was shown in fig. 1.

To estimate the maximum value of the relative thickening of the inner edge of the annular disk$(\Delta H/H_0)$, equating displacement volumes assuming a linear dependenceΔH on the current radiusρ, we obtain:

$$\frac{\Delta H}{H_0}=\frac{r_0^2}{R^2}\frac{(\rho_0^2/r_0^2-1)}{(1-\rho_0^2/R^2)} \qquad (20)$$

Numerical calculation with the parameters $\rho_0/r_0=1$and $R/\rho_0=2$ shows that the relative axial thickening of the inner edge,$H/H_0\cong 0.14$ which is comparable with the results of experimental studies.

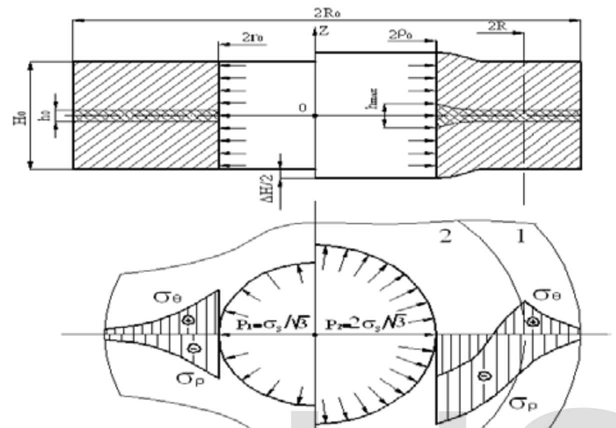

Fig. 1. represented the initial (left) and final (right) shape and dimensions of the unit, as well as the stress distribution at the initial and the limiting value of internal pressure (1-elastic region, 2-plastic region).

3 Conclusion

1. The problem of tube hydro forming is solved in the framework of the theory of plastic flow through the membrane theory of shells. It is shown that the initial equations of plastic plane stress with axial symmetry deformation are reflected on the deviatory plane (π-plane) of the cylinder of plasticity in the form of a simple differential dependence between the increments of the radial stress and equivalent strain.
2. It is shown that the interrelated changes of thickness and strain hardening can obtain an analytical model of the process of forming only with a proportional change in the stress components.
3. It is shown that in the forming process of thin-walled cylindrical shells the limiting factor of forming (LFF) for the ideally rigid-plastic model of a deformable material equals 2,35.
4. An analytical solution to the problem of forming annular disk of constant thickness at large plastic deformation, taking into account interrelated changes in the effective strain and strain hardening.
5. It is shown that the relative thickening of the inner contour of the disk in the axial direction does not exceed 15%.
6. The present model can be useful in conducting parametric studies on the different parameters affecting the process including die design, process and material parameters.

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